Evaluate the following integrals, or explain why they can't be evaluated. **SCORE: _____ / 11 PTS**

[a] $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin r\cos r}{1+\cos^6 r}\,dr$ $= 0$

$\dfrac{\sin(-r)\cos(-r)}{1+\cos^6(-r)} = \dfrac{-\sin r\cos r}{1+\cos^6 r}$ (1)

ODD, CONTINUOUS

ALL ITEMS WORTH $\frac{1}{2}$

EXCEPT AS INDICATED

[b] $\displaystyle\int \frac{(2y^2-3\sqrt{y})^2}{6y^5}\,dy$

$= \displaystyle\int \frac{4y^4-12y^{\frac{5}{2}}+9y}{6y^5}\,dy$

$= \displaystyle\int \left(\frac{2}{3}y^{-1}-2y^{-\frac{5}{2}}+\frac{3}{2}y^{-4}\right)dy$ (1)

$= \frac{2}{3}\ln|y| - 2\left(-\frac{2}{3}\right)y^{-\frac{3}{2}} + \frac{3}{2}\left(-\frac{1}{3}\right)y^{-3} + C$

$= \frac{2}{3}\ln|y| + \frac{4}{3}y^{-\frac{3}{2}} - \frac{1}{2}y^{-3} + C$

[c] $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sin\theta\cos\theta}{\sqrt{1-\cos^4\theta}}\,d\theta$

$u = \cos^2\theta$ (1)

$du = -2\cos\theta\sin\theta\,d\theta$

$-\frac{1}{2}du = \cos\theta\sin\theta\,d\theta$

$\theta = \frac{\pi}{3} \to u = \frac{1}{4}$

$\theta = \frac{\pi}{2} \to u = 0$

(1) $\displaystyle\int_{\frac{1}{4}}^{0} -\frac{1}{2}\frac{1}{\sqrt{1-u^2}}\,du$ (1)

$= -\frac{1}{2}\sin^{-1}u \Big|_{\frac{1}{4}}^{0}$

$= -\frac{1}{2}\left(0-\sin^{-1}\frac{1}{4}\right)$

$= \frac{1}{2}\sin^{-1}\frac{1}{4}$

[d] $\displaystyle\int_{-\pi}^{\pi} \frac{t+\sin t}{\cos t}\,dt$

DISCONTINUOUS @ $t = \pm\frac{\pi}{2}$ (1)

FTC DOESN'T APPLY

If $p(x) = \int_{x^2}^{5x} \tan^{-1} e^{4t}\, dt$, find $p'(x)$.

SCORE: _____ / 4 PTS

$$p(x) = \int_{x^2}^{0} \tan^{-1} e^{4t}\, dt + \int_0^{5x} \tan^{-1} e^{4t}\, dt$$

$$= -\int_0^{x^2} \tan^{-1} e^{4t}\, dt + \int_0^{5x} \tan^{-1} e^{4t}\, dt$$

$$p'(x) = \frac{d}{dx} - \int_0^{x^2} \tan^{-1} e^{4t}\, dt + \frac{d}{dx} \int_0^{5x} \tan^{-1} e^{4t}\, dt$$

$$= -\frac{d}{d(x^2)} \int_0^{x^2} \tan^{-1} e^{4t}\, dt \cdot \frac{d(x^2)}{dx} + \frac{d}{d(5x)} \int_0^{5x} \tan^{-1} e^{4t}\, dt \cdot \frac{d(5x)}{dx}$$

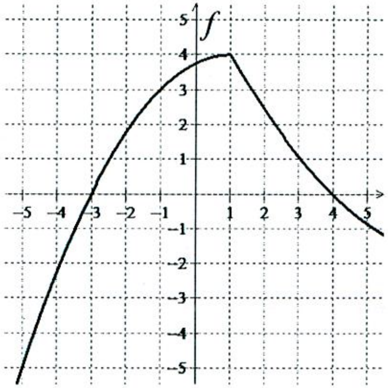
$$= -\tan^{-1} e^{4x^2} \cdot 2x + \tan^{-1} e^{20x} \cdot 5$$

$$= 5 \tan^{-1} e^{20x} - 2x \tan^{-1} e^{4x^2}$$

ALL ITEMS WORTH 1

Let $g(x) = \int_{-2}^{x} f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 7 PTS**

[a] Write "I UNDERSTAND THAT THE GRAPH SHOWS f , BUT THE QUESTIONS ASK ABOUT g ".



[b] Find $g'(-1)$. **Explain your answer very briefly.**

$g'(-1) = \underbrace{f(-1) = 3}$

ALL ITEMS WORTH 1 EXCEPT AS INDICATED

[c] Find all critical numbers of g . **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) = 0}$ @ $\underbrace{x = -3, 4}_{2}$

[d] Find all intervals over which g is both increasing and concave down at the same time. **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) > 0}$ AND $\underbrace{\text{DECREASING}}$ ON $\underbrace{(1,4)}$

In complete sentences, using proper English and mathematical notation, state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

IF f IS CONTINUOUS ON $[a, b]$

① IF $g(x) = \int_a^x f(t)\,dt$, THEN $g'(x) = f(x)$

② IF $F'(x) = f(x)$, THEN $\int_a^b f(t)\,dt = F(b) - F(a)$

IF F' IS CONTINUOUS ON $[a, b]$

THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

GRADED BY ME

If $f(a)$ is the annual amount of weight Morgan gained (in kilograms per year) when she was a years old, **SCORE: _____ / 3 PTS**

what is the meaning of the equation $\int_{12}^{14} f(x)\,dx = 16$?

NOTES: **Your answer must use all three numbers from the equation, along with correct units.**
Your answer should NOT use "a", "x", "f(x)", "integral", "antiderivative", "rate of change" or "derivative".
Your answer should sound like normal spoken English.

FROM AGE 12 YEARS OLD TO 14 YEARS OLD,
MORGAN'S WEIGHT INCREASED 16 kg ALTOGETHER.

GRADED BY ME